Available online at www.sciencedirect.com
ScienceDirect
Procedia Engineering 144 (2016) 584 – 591
Procedia Engineering
www.elsevier.com/locate/procedia

12th International Conference on Vibration Problems, ICOVP 2015

Linear and Nonlinear Analysis of Piezoelectric Based Vibration Absorber with Acceleration Feedback

S Mohanty*, S K Dwivedy

Indian Institute of Technnology Guwahati, Department of Mechancial Engineering, Assam, Guwahati, 781039, India

Abstract

In the present paper, analysis of a single degree of freedom spring-mass-damper primary system with a piezoelectric based dynamic vibration absorber (DVA) is carried out. The proposed DVA model consisting of a lead zirconate titanate (PZT) actuator which is connected in series with a spring. The analysis is done in two section by considering a static force and a harmonic force acting on the primary system respectively. In the first section linear stiffness is considered in the primary mass and in the second section cubic nonlinear stiffness along with the linear stiffness is considered. The voltage applied to the stack PZT actuator is considered to be proportional to the acceleration of the primary mass. Method of multiple scales (MMS) is used to obtain the system response in the nonlinear analysis and compared with linear analysis. For linear system optimum system parameters of the absorber are obtained using fixed point theory of optimization and Routh's stability criterion is used for stability analysis. Primary resonance condition is studied in the nonlinear analysis. In the proposed model as a spring is connected in series with PZT actuator for which force developed by the absorber to suppress the vibration of the primary system is not solely depend upon the voltage, so one can use small voltage to reduce the vibration of primary system.

Peer-review under responsibility of the organizing committee of ICOVP 2015

Keywords: *Tuned mass damper, Piezoelectric actuator, Method of multiple scales, Fixed point theory.*

1. Introduction

Dynamic vibration absorber (DVA) have been well researched over the past decades [1] showing various applications in the field of engineering systems. But still suppression of vibration by the DVA is an active area of research as large number of recent literature review papers [2-8] are available in this field. Various optimization techniques [9, 10] have been developed to minimize the peak amplitude of the primary vibrating system by attaching

* Corresponding author. Tel.: +91-9678436595;
E-mail address: siba.mech@gmail.com

Peer-review under responsibility of the organizing committee of ICOVP 2015
doi:10.1016/j.proeng.2016.05.045

the tuned mass damper to the primary system. Different vibration absorption models [11-14] have been developed to reduce the peak and valley height of the frequency response curve and also maximum amplitude of the primary system. Many vibration absorber recently use various smart material like piezoelectric in stack and patch form to mitigate the vibration of the single degree of freedom (DOF) multi DOF and in continuous system and also harvesting energy from the system [15-17]. These vibration absorber are known as Active vibration absorber which uses sensor and actuator to provide counteracting force to the vibrating primary system to reduce to its vibration. Ji [18] used MMS for analyzing and studying different resonance condition of a nonlinear passive vibration absorber (PVA) with a nonlinear SDOF system vibrating with harmonic excitation. Nonlinear analysis of various absorber model with SDOF and MDOF are also studied in [19, 20] by considering multiple harmonic force and parametric excitation forces acting on the primary system. However the nonlinear analysis of the active vibration absorber with PZT actuator is not explored more. In the proposed model the PZT actuator is connected in series with the absorber by a spring which are not present in the previous literatures. So by this type of model one can use a higher order stiffness value for the absorber to produce more controlling force without much increase in the voltage provided to the actuator. This work will be an extension of [21] where linear analysis is carried out and nonlinear analysis with displacement feedback is done.

2. Mathematical Modelling

A non-linear single degree of freedom primary system to which an active DVA is placed is shown in Fig. 1. (a) The block diagram of the acceleration feedback on the primary system is shown in Fig. 1. (b). Here m_i, c_i and k_i denotes mass, damping and stiffness of the primary system and the DVA respectively for i =1, 2. k_3 and k_E^P denotes the stiffness of the absorber and piezoelectric actuator respectively. $F(t)$ is the force acting on the primary system. x_1 and x_2 are the displacement produced in the primary mass and absorber respectively.

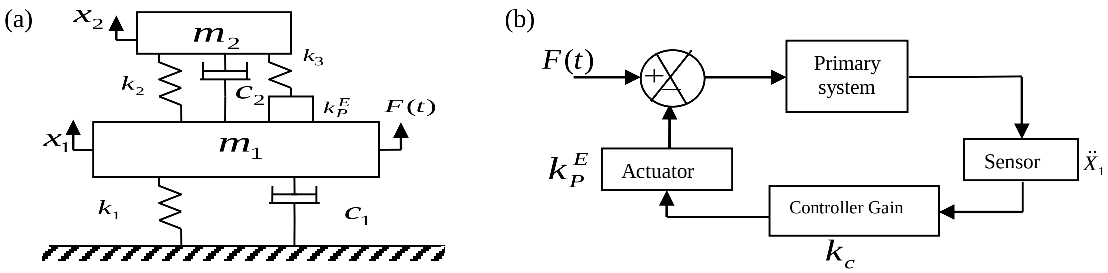

Fig. 1. (a) Piezoelectric stack actuator based Hybrid vibration absorber; (b) block diagram for acceleration feedback of the primary mass.

2.1. Linear system analysis

The equations of motion of the system in the Fig. 1. (a) can be compared with [9] but here as a spring of stiffness k_3 is attached in tandem with the PZT actuator of stiffness of k_P^E the actuating force developed will be different than [9]. So the modified equations of motion can be written as

$$m_1 x_1'' + k_1 x_1 + k_2 (x_1 - x_2) + c_1 x_1' + c_2 \left(x_1' - x_2'\right) = F(t) - k_r (x_1 + \delta_0 - x_2), \tag{1}$$

$$m_2 x_2'' + k_2 (x_2 - x_1) + c_2 \left(x_2' - x_1'\right) = k_r (x_1 + \delta_0 - x_2), \tag{2}$$

where $k_r = \left(k_P^E k_3 / \left(k_P^E + k_3\right)\right)$ and $\delta_o = n d_{33} v$

The nominal displacement of the stack actuator [10] is δ_o, where n is the number of wafer used in the stack actuator

ν is the applied voltage and d_{33} is dielectric charge constant.
Recasting equations (1) and (2) into respective non-dimensional forms one writes,

$$\ddot{X}_1 + X_1 + 2\xi_1 \dot{X}_1 - \mu \Omega_2{}^2 X - 2\xi_2 \mu \Omega_2 \dot{X} = f(\tau) - k\lambda\upsilon \tag{3}$$

$$\ddot{X} + \Omega_2{}^2 X + 2\xi_2 \Omega_2 \dot{X} = -\ddot{X}_1 + (k\lambda\upsilon / \mu). \tag{4}$$

The non-dimensional parameters used in Eq. (3) and Eq. (4) are

$$X_1 = (x_1 / x_o), X_2 = (x_2 / x_o), X = X_2 - X_1, k = (k_r / k_1), \mu = (m_2 / m_1), \xi_2 = (c_2 / 2m_2\omega_2), \xi_1 = (c_1 / 2m_1\omega_o),$$
$$\Omega_2 = (\omega_2 / \omega_0), \lambda = (nd_{33}v_o / x_o), \upsilon = (v / v_o), \omega_o = \sqrt{(k_1 / m_1)}, \omega_2 = \sqrt{(k_2 + k_r) / m_2}$$

x_0 and v_0 are reference displacement and voltage quantity. The 'dot' denotes differentiation with respect to the non-dimensional time $\tau = \omega_0 t$. Considering the effect of acceleration feedback of the primary mass is analysed, where as in [4] displacement and velocity feedback were considered. The control block diagram of the system is shown in Fig. 1. (b), with control gain 'k_c' and accordingly the control law is written as $\upsilon = -k_c \ddot{X}_1$. Taking Laplace transforms to the Eq. (3) and Eq. (4), the frequency response of the transfer function of the primary mass is obtained as

$$G(j\omega) = \frac{\Omega_2{}^2 - \omega^2 + j2\xi_2\Omega_2\omega}{(b_4{}^4\omega^4 - b_2\omega^2 + b_0) + j(b_1\omega - b_3\omega^3)} \tag{5}$$

where the coefficients b_4, b_3, b_2, b_1 and b_0 are expressed as

$$\alpha = k\lambda k_c, b_4 = 1 - \alpha, b_3 = 2\xi_2\Omega_2 + 2\xi_1 + 2\xi_2\Omega_2\mu, b_2 = \Omega_2{}^2 + 4\xi_1\xi_2\Omega_2 + \mu\Omega_2{}^2 + 1, b_1 = 2\xi_1\Omega_2{}^2 + 2\xi_2\Omega_2, b_0 = \Omega_2{}^2$$

Obtained the optimal tuning ratio, damping ratio by using H_∞ optimization technique as $\Omega_2 = \sqrt{(1-\alpha)/(1+\mu)^2}$ and the optimum damping ratio as referred from [9]. The stability of the system is found by using Routh's stability criterion, which showed for the particular range of the control force $0 < \alpha < 1$ the system is stable where the roots of the characteristic Eq. (5) is negative real and complex.

2.2. Nonlinear Analysis

In this section nonlinear analysis has been carried out by considering additional non-dimensional cubic nonlinear stiffness of k_{11} present in the primary system and harmonic force $f\cos(\Omega\tau)$ is acting on it. So the modified non-dimensional Eq. (3) and Eq. (4) can be written as

$$\ddot{X}_1 + X_1 + 2\xi_1\dot{X}_1 + \mu\Omega_2{}^2 X_1 - \mu\Omega_2{}^2 X_2 + 2\xi_2\mu\Omega_2\dot{X}_1 - 2\xi_2\mu\Omega_2\dot{X}_2 + k_{11}X_1{}^3 = f\cos(\Omega\tau) - k\lambda\upsilon \tag{6}$$

$$\ddot{X}_2 - \Omega_2{}^2 X_1 + \Omega_2{}^2 X_2 - 2\xi_2\Omega_2\dot{X}_1 + 2\xi_2\Omega_2\dot{X}_2 = (k\lambda\upsilon / \mu) \tag{7}$$

Considering acceleration feedback on the primary system and accordingly control law is written as $\upsilon = -k_c\ddot{X}_1$. Substituting this value and rearranging the reduced non-dimensional by considering book keeping parameter 'ε' the modified equations motion can be written as.

$$\ddot{X}_1+\omega_0^2X_1+\varepsilon\hat{\xi}_1\dot{X}_1+\varepsilon\hat{k}X_1^3-\varepsilon^2\hat{\mu}X_2-\varepsilon^2\hat{\xi}_2\dot{X}_2=\varepsilon F\cos\Omega\tau \tag{8}$$

$$\ddot{X}_2+\Omega_2^2X_2+\varepsilon\varsigma\dot{X}_2=-(\alpha/\mu)\ddot{X}_1+2\xi_2\Omega_2\dot{X}_1+\Omega_2^2X_1 \tag{9}$$

Analysing the right part of the Eq. (9) developed due to the applied counteracting force on the primary system. These right hand side terms which consists of inertia, damping and displacement term of the primary system can be considered as forcing term. Assuming this force to be a combination of harmonic force and parametrical excited force and the corresponding equation can be written as

$$\ddot{X}_2+\Omega_2^2X_2+\varepsilon\varsigma\dot{X}_2=\varepsilon F_2\cos\Omega_1\tau+\varepsilon F_3X_1\cos\Omega_1\tau \tag{10}$$

The forcing term F_2 and F_3 are evaluated by first solving the Eq. (8) then substituting the values of X_1 in the Eq. (9) where the non-dimensional parameters are

$$\alpha=k\lambda k_c,\omega_0=\sqrt{\left(1+\mu\Omega_2^2\right)/(1-\alpha)},\hat{\mu}=\mu\Omega_2^2/\varepsilon^2(1-\alpha),\hat{\xi}_1=(2\xi_1+2\xi_2\mu\Omega_2)/\varepsilon(1-\alpha)$$
$$\hat{\xi}_2=2\xi_2\mu\Omega_2/\varepsilon^2(1-\alpha),\hat{k}=k_{11}/\varepsilon(1-\alpha),F=f/(1-\alpha),\varsigma_2=2\xi_2\Omega_2$$

The 'dot' denotes differentiation with respect to the non-dimensional time $\tau=\omega_0 t$. MMS is used to obtain the approximate solutions of the Eq. (8) and Eq. (9) by considering the perturb solution as $X_1=X_{10}(\tau_0,\tau_1)+\varepsilon X_{11}(\tau_0,\tau_1)$ and $X_2=X_{20}(\tau_0,\tau_1)+\varepsilon X_{21}(\tau_0,\tau_1)$ The time scales can be written as $\tau_n=\varepsilon^n t$, with n =0, 1, 2,...Time derivatives along different time scales lead to the differential operators $d/dt=D_0+\varepsilon D_1$ and $d^2/dt^2=D_0^2+2\varepsilon D_0D_1$ Substituting the proposed solutions $X_1(\tau_0,\tau_1)$ and $X_2(\tau_0,\tau_1)$ into Eq. (8) and Eq. (10) and equating same power of ε yields the following set of partial differential equations are obtained.

$$\varepsilon^0:\left(D_0^2+\omega_0^2\right)X_{10}=0 \quad (11) \quad \text{and} \quad \left(D_0^2+\Omega_2^2\right)X_{20}=0 \tag{12}$$

$$\varepsilon^1:\left(D_0^2+\omega_0^2\right)X_{11}=-2D_0D_1X_{10}-\hat{\xi}_1D_0X_{10}-\hat{k}X_{10}^3+F\cos(\Omega\tau_0) \tag{13}$$

$$\left(D_0^2+\Omega_2^2\right)X_{21}=-2D_0D_1X_{20}-\varsigma_2D_0X_{20}+F_2\cos(\Omega_1\tau_0)+F_3X_{10}\cos(\Omega_1\tau_0) \tag{14}$$

The solution of Eq. (11) and Eq. (12) may be written as follows

$$X_{10}=A_1(\tau_1)\exp(i\omega_0\tau_0)+cc \quad (15) \quad \text{and} \quad X_{20}=A_2(\tau_1)\exp(i\Omega_2\tau_0)+cc \tag{16}$$

where A_1 and A_2 are unknown complex functions of non-dimensional time τ_1 and 'cc' stands for complex conjugate of preceding terms. Substituting Eq. (15) and Eq. (16) into Eq. (13) and Eq. (14) by considering primary resonance condition i.e. when $\Omega=(\omega_0+\varepsilon\sigma_1)\tau_0$ and $\Omega_1=(\Omega_2+\varepsilon\sigma_2)\tau_0$ the corresponding equation is modified as

$$\left(D_0^2+\omega_0^2\right)X_{12}=-2D_1\left(i\omega_0A_1e^{i\omega_0\tau_0}\right)-\hat{\xi}_1\left(i\omega_0A_1e^{i\omega_0\tau_0}\right)-\hat{k}\left(A_1e^{i\omega_0\tau_0}+cc\right)^3+\left(\left(Fe^{i(\omega_0\tau_0+\sigma_1\tau_1)}\right)/2\right) \tag{17}$$

$$\left(D_0{}^2+\Omega_2{}^2\right)X_{21}=-2i\Omega_2 D_1 A_2 e^{i\Omega_2\tau_0}-\varsigma_2 i\Omega_2 A_2 e^{i\Omega_2\tau_0}+\left(\left(F_2 e^{i(\Omega_2+\varepsilon\sigma_2)\tau_0}\right)/2\right)+\left(\left(F_3 A_1 e^{i(\omega_0+\Omega_2+\varepsilon\sigma_2)\tau_0}\right)/2\right) \tag{18}$$

obtaining the secular terms leads to the following differential equation

$$2D_1A_1 i\omega_0=-\hat{\xi}_1 i\omega_0 A_1-3\hat{k}A_1{}^2\bar{A}_1+\left(Fe^{i\sigma_1\tau_1}/2\right) \quad (19) \quad \text{and} \quad 2D_1A_2 i\Omega_2=-\varsigma_2 i\Omega_2 A_2+\left(F_2 e^{i\sigma_2\tau_1}/2\right) \quad (20)$$

eliminating secular term the Eq. (21) and Eq. (22) reduces to

$$X_{11}=\left(\hat{k}A_1{}^3 e^{3i\omega_0\tau_0}/8\omega_0{}^2\right)+cc \quad (21) \quad \text{and} \quad X_{21}=\left(F_3A_1 e^{i(\omega_0+\Omega_2+\varepsilon\sigma_2)\tau_0}/2\right)+cc \quad (22)$$

The amplitude functions A_1 and A_2 may be expressed in polar form as $A_2=\frac{1}{2}a_1 e^{i\beta_1}$, $A_2=\frac{1}{2}a_2 e^{i\beta_2}$. Where a_1, a_2, β_1 and β_2 are real functions of the time τ_1 and τ_2, which may be transformed into autonomous system by letting $\gamma_1=\sigma_1\tau_1-\beta_1$ and $\gamma_2=\sigma_2\tau_2-\beta_2$. Substituting A_1 and A_2 into Eq. (19) and Eq. (20) and separating real and imaginary part leads to following sets of equation.

$$a_1\gamma_1{}'=\sigma_1 a_1-\left(3\hat{k}_1 a_1{}^3/8\omega_0\right)+\left(F\cos\gamma_1/2\omega_0\right) \quad (23) \quad \text{and} \quad a_1{}'=-\left(\hat{\xi}_1 a_1/2\right)+\left(F\sin\gamma_1/2\omega_0\right) \quad (24)$$

$$a_2\gamma_2{}'=\sigma_2 a_2+\left(F_2\cos\gamma_2/2\Omega_2\right) \quad (25) \quad \text{and} \quad a_2{}'=-\left(a_2\varsigma_2/2\right)+\left(F\sin\gamma_2/2\Omega_2\right) \quad (26)$$

The steady state response of the system are computed by making $a_1{}'$, $a_2{}'$, $\gamma_1{}'$ and $\gamma_2{}'$ = 0. Solving the equations amplitude response as function of detuning parameter σ_1 may be written as

$$\sigma_1=\left(3\hat{k}a_1{}^2/8\omega_0\right)\pm\sqrt{\left(\left(F^2/4\omega_0{}^2a_1{}^2\right)-\left(\hat{\xi}_1{}^2/4\right)\right)} \quad (27) \quad \text{and} \quad \sigma_2=\pm\sqrt{\left(\left(F^2/4\Omega_2{}^2a_2{}^2\right)-\left(\varsigma_2{}^2/4\right)\right)} \quad (28)$$

Hence the solution of the Eq. (8) can be written as

$$X_1=a_1\cos(\Omega\tau-\gamma)+\varepsilon\left(\hat{k}a_1{}^3/32\omega_0\right)\cos3(\Omega\tau-\gamma)+O\left(\varepsilon^2\right) \tag{29}$$

Substituting the value of X_1 in the Eq. (9) F_2 is determined as $\left(\left(\varepsilon\alpha\Omega^2\right)/\mu+\Omega_2{}^2\right)a_1$ and F_3 as $\left(\left(3\varepsilon\alpha\Omega_2{}^2\Omega\right)/16\omega_0+\right)a_1{}^3$.

3. Results and discussions

3.1. Linear system analysis

Considering the ratio of secondary mass to primary mass μ equal to 0.05 the numerical analyses have been carried in this work. The damping parameter of the primary system is taken to be zero. The effect of the control force α on the frequency response characteristics of the primary system is shown in the Fig. 2, which shows that as the value of α increases the vibration amplitude is reduces. When α is taken as 0.0001 amplitude is found to be 6.45 times the static deflection of the primary mass as shown in the dotted line. This can happen when there will be some electronic malfunction occur in the system but due to the optimum parameter of the absorber the resonating vibration amplitude will not very large. But when α taken as 0.99 amplitude reduces to 1.15 times the static deflection of primary mass within the stability region in the broader frequency region as shown in thick line. The optimum damping ratio ξ_2 in the absorber is found to be 0.474. It is observed that if the optimum damping ratio parameter is not taken then primary system amplitude is very large irrespective of the increasing damping ratio of the absorber when there is no applied controlling force. So it is suggested to use optimum value for suppressing the vibration of primary system.

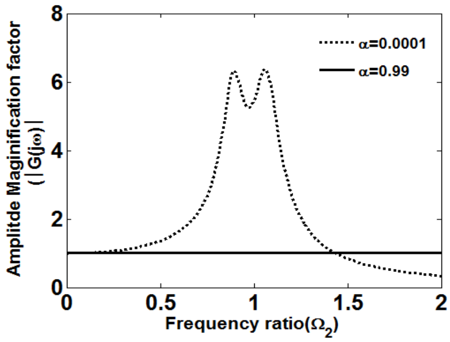

Fig. 2. Frequency-response plots of the primary system with acceleration feedback of the primary mass by linear analysis.

3.2. Nonlinear system analysis

Fig. 3. (a) shows the effect of the amplitude of the primary mass with addition cubic nonlinearity as in Eq. (27) with damping ratio of the primary mass taken as 0.04 while all other parameters are kept same as the linear analysis. The observation are taken when α is 0.0001and 0.99. It can be observed that when α is 0.0001 then the amplitude is very high as shown in dotted line and reduces to 0.106 when the α increases to 0.99 as shown in thick line. Fig. 3. (b) shows when the nonlinearity used in the spring stiffness is increased, which bends the frequency response curve showing hardenig effect due to which one can observe multiple solution and stability and unstability region. The time responses for primary system is shown in Fig. 3. (c) which is obtained using Eq. (6) and Eq. (7). For obtaining the response optimum parameter and controlling parameter as obtained in the formulation is used. It is observed that amplitude of displacement response decrease with time with and without the application of controlling force α . But the settling time of the primary system by the application of α is faster and it is around 14 non-dimensional time as

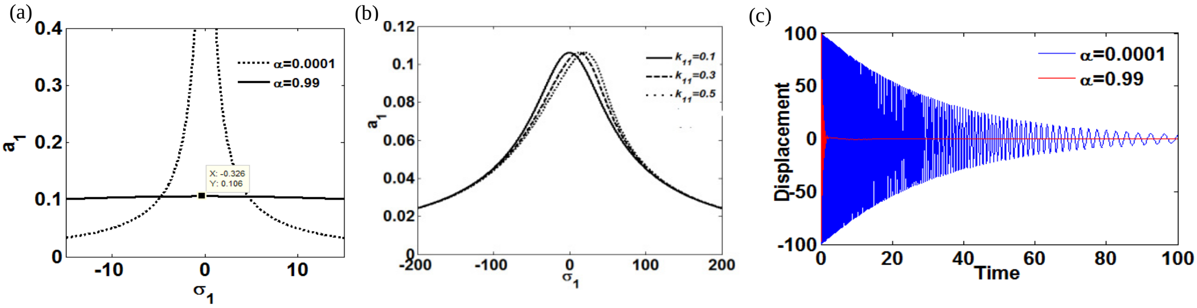

Fig. 3. Frequency response curves for primary system (a) different α (b) different nonlinearity (c) time domain response curve.

shown in red line to 0.05 non dimensional amplitude whereas without controlling force its takes much longer time to settle as shown in blue line. From Fig. 3 (c). it can be observed that the response obtained by using the method of multiple scales is in good agreement with the numerically solved temporal equation of motion. In Fig. 4. frequency response of the absorber is analysed under primary resonance condition. It can be observed that as α increases the corresponding amplitude of absorber also increases. When α of 0.0001 is applied the amplitude of the absorber is 0.56 whereas when α of 0.99 is appllied than the amplitude is incresed to 1.32.

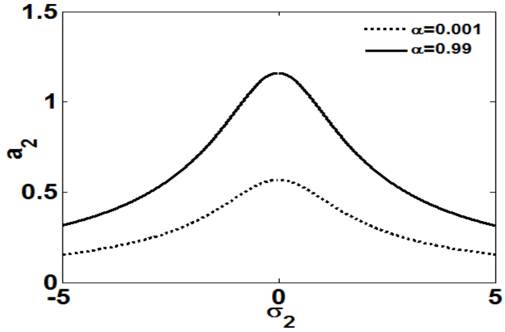

Fig. 4. Frequency response curves of the absorber with different values of α .

4. Conclusions

In the present paper, active vibration absorber with acceleration feedback for both linear and nonlinear analysis is investigated by considering a new model. The proposed model comprises a conventional dynamic vibration absorber in tandem with a PZT stack actuator to provide the active control force. Optimum parameters for the absorber configuration obtained using fixed point theory and Routh's stability criterion is used for stability analysis. From the linear analysis it is found that the amplitude of the primary system without active force is 6.45 to the static deflection but its amplitude reduces to 1.15 times the static deflection when controlling force is applied within the stability region of operation. Nonlinear analysis also investigated by considering a cubic nonlinear stiffness along with the linear stiffness in the primary system. In the nonlinear analysis the amplitude of the primary system is found to be 0.106 for the primary resonance condition when harmonic force is acting on it. Time domain analysis of the primary system is carried out showing effectiveness vibration suppression by the application of controlling force. In the proposed model as a spring of stiffness is used in series with the actuator so one can increase the controlling force without increasing the piezoelectric stiffness or the voltage parameter, so the purposed model is more economical as controlling force parameter is not merely depend upon voltage regulation and also the model is fail-safe design. When the control is on, the system performs better than the passive DVA. However, in events of failure of the active part, the optimal passive absorber can still protect the system from being damaged due to severe vibration.

References

[1] H. A Frahm, Device for damping vibrations of bodies, US Patent (1911) 989958.
[2] M. G. Soto, H. Adeli, Tuned mass dampers, Archives of Computational Methods in Engineering, 20.4 (2013) 419-431.
[3] O. El-Khoury, H. Adeli, Recent advances on vibration control of structures under dynamic loading. Archives of Computational Methods in Engineering 20.4 (2013) 353-360.
[4] N. R. Fisco, H. Adeli, Smart structures: part I—active and semi-active control. Scientia Iranica, 18.3 (2011) 275-284.
[5] W. Ya, D. J. Inman, A survey of control strategies for simultaneous vibration suppression and energy harvesting via piezoceramics. Journal of Intelligent Material Systems and Structures 23.18 (2012) 2021-2037.
[6] N. R. Fisco, H.Adeli, Smart structures: part II—hybrid control systems and control strategies, Scientia *Iranica*, 18.3 (2011) 285-295.
[7] S. Korkmaz, A review of active structural control: challenges for engineering informatics, Computers & Structures, 89.23 (2011) 2113-2132.
[8] L. Kela, P. Vähäoja, Recent studies of adaptive tuned vibration absorbers/neutralizers, ASME, Applied Mechanics Review, 62.6 (2009) 1–9.
[9] Y. L. Cheung, W. O. Wong, L. Cheng, Design optimization of a damped hybrid vibration absorber, Journal of sound and vibration 331.4, (2012) 750-766.
[10] S.Chun, Y. Lee, T.H. Kim, H_{∞} optimization of dynamic vibration absorber variant for vibration control of damped linear systems, Journal of Sound and Vibration, 335 (2015) 55-65.
[11] S. Krenk, J. Høgsberg, Tuned mass absorber on a flexible structure, Journal of Sound and Vibration 333.6 (2014) 1577-1595.
[12] P. Xiang, A. Nishitani, Optimum design for more effective tuned mass damper system and its application to base-isolated buildings, Structural Control and Health Monitoring 21.1 (2014) 98-114.

[13] J. S. Issa, Optimal design of a damped single degree of freedom platform for vibration suppression in harmonically forced undamped systems, Journal of Vibration and Acoustics 135.5 (2013) 051003.
[14] M. Silveira, B. R. Pontes Jr., J. M. Balthazar, Use of nonlinear asymmetrical shock absorber to improve comfort on passenger vehicles, Journal of Sound and Vibration 333.7 (2014) 2114-2129.
[15] C. Sun, R. P. Eason, S. Nagarajaiah, A. J. Dick, Hardening Duffing oscillator attenuation using a nonlinear TMD a semi-active TMD and multiple TMD, Journal of sound and vibration 332 (2013) 674-686.
[16] Y. L. Cheung, W. O. Wong, L. Cheng, Optimization of a hybrid vibration absorber for vibration control of structures under random force excitation, Journal of Sound and Vibration 332.3 (2013) 494-509.
[17] F. An, W. Chen, M. Shao, Dynamic behaviour of time-delayed acceleration feedback controller for active vibration control of flexible structures, Journal of Sound and Vibration 333.20 (2014) 4789-4809.
[18] J. C. Ji, Design of a nonlinear vibration absorber using three-to-one internal resonances, Mechanical Systems and Signal Processing 42.1 (2014) 236-246.
[19] E. E. El-Behady, E. R El-Zahar., Vibration reduction and stability study of a dynamical system under multi-excitation forces via active absorber, International Journal of Physical Sciences 7.48 (2013) 6203-6209.
[20] Y. A. Amer, M. Sayed, Stability at Principal Resonance of Multi-Parametrically and Externally Excited Mechanical System, Advances in Theoretical and Applied Mechanics 4.1 (2011) 1-14.
[21] S. Mohanty, S. Chatterjee, S. K. Dwivedy, Analysis of Hybrid Vibration Absorber with Piezoelectric Actuator, Proceedings of the Sixth International Conference on Theoretical, Applied, Computational and Experimental Mechanics (ICTACEM 2014), IIT Kharagpur, India (2014) 2014. ISBN: 978-93-80813-30-1.